



CSS Past Papers

Subject: Pure Mathematics

Year: 2023

CSS/PMS GRAMMAR

— علم بڑی طاقت ہے —



FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2023 FOR RECRUITMENT
TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS

MAXIMUM MARKS = 100

- NOTE:** (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C**. **ALL** questions carry **EQUAL** marks.
- (ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
- (iii) Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
- (iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
- (v) Extra attempt of any question or any part of the attempted question will not be considered.
- (vi) **Use of Calculator is allowed.**

SECTION-A

- Q. 1. (a) Find centre of S_3 . (10)
- (b) Using the row operations, show that the matrix $\begin{pmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{pmatrix}$ has no inverse. (10) (20)
- Q. 2. (a) For any group G , show that $\frac{G}{\{e\}} \cong G$ and $\frac{G}{G} \cong \{e\}$. (10)
- (b) Suppose U and W are distinct four dimensional subspaces of a vector space V of dimension six. Find the possible dimension of $U \cap W$. (10) (20)
- Q. 3. (a) For what value of α is the matrix $\begin{pmatrix} -\alpha & \alpha-1 & \alpha+1 \\ 1 & 2 & 3 \\ 2-\alpha & \alpha+3 & \alpha+7 \end{pmatrix}$ is singular? (10)
- (b) Define $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ by $T(x_1, x_2, x_3) = (-x_3, x_1, x_1 + x_3)$. Find $N(T)$. Is T one-to-one? (10) (20)

SECTION-B

- Q. 4. (a) Find the value of θ and the limit in order that $\lim_{x \rightarrow 0} \frac{\sin 2x + \theta \sin x}{x^3}$ be finite. (10)
- (b) Show that $x < \sin^{-1} x < \frac{x}{\sqrt{1-x^2}}$, $0 < x < 1$. (10) (20)
- Q. 5. (a) Given that $U = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$. Verify that $U_{xx} + U_{yy} + U_{zz} = 0$. (10)
- (b) Evaluate $\iint (x^2 + y^2) dx dy$, over the domain bounded by $y = x^2$ and $x = y^2$. (10) (20)
- Q. 6. (a) Evaluate $\iint (x^2 + y^2) dx dy$, over the region bounded by $xy=1$, $y=0$, $y=x$ and $x=2$. (10)
- (b) Find an equation of a normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ in the form $ax \cos \theta + by \cot \theta = a^2 + b^2$. Prove that the normal is external bisector of the angle between the focal distances of its foot. (10) (20)



SECTION-C

Q. 7. (a) Determine k such that $U = e^{2x} \cos ky$ is harmonic and find a conjugate harmonic. (10)

(b) Evaluate $\int_C (\frac{1}{z^5} + z^3) dz$ from 1 to -1 along the upper arc of the unit circle. (10) **(20)**

Q. 8. (a) Find the Laurent Series of $\frac{1}{1-z^2}$ in the region $0 < |z-1| < 2$. (10)

(b) Find the residues at the singular points of $\frac{-Z^2 - 22z + 8}{Z^3 - 5z^2 + 4z}$ which lie inside the circle $|z|=2$. (10) **(20)**



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